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MONETARY SYSTEMS AND ACCELERATOR MODELS

By HYMAN P. MINSKY*

A significant part of recent literature on both growth and business-cycle theory has been based upon some form of an interaction between a consumption (saving) relation and an induced investment relation. The authors who have constructed these accelerator-multiplier models have paid little, if any, attention to the monetary prerequisites and effects of the assumed processes.[1] Obviously the accelerator-multiplier process takes place in the context of some monetary system. In this paper the manner in which the time series generated depends upon the interaction of an accelerator-multiplier process and the monetary system will be investigated: the main emphasis will be on the upper turning point and the possibility of generating steady growth. In this paper the lower turning point is unexplained aside from noticing how the various monetary systems can act as a brake on disinvestment and also, by changing liquidity, set the stage for a recovery.

The procedure will be to examine the result of combining a linear accelerator-multiplier model with a number of alternative monetary systems. The terms (interest rate) and the manner (type of liability) of financing investment are affected by the behavior of the monetary system. In turn, both money-market conditions and the balance-sheet structure of firms affect the response of firms to a change in income. This can be interpreted as making the accelerator coefficient an endogenous variable related to the monetary system. Hence the material

* The author is associate professor of economics, Brown University. A large portion of the work on this paper was done while he was a visiting associate professor at the University of California, Berkeley. He wishes to acknowledge his debt to Julius Margolis, Roger Miller and Merton P. Stoltz for their helpful comments and suggestions.

[1] J. R. Hicks, *A Contribution to the Theory of the Trade Cycle* (Oxford, 1950) and S. C. Tsiang, "Accelerator, Theory of the Firm, and the Business Cycle," *Quart. Jour. Econ.*, Aug. 1951, LXV, 325–41 briefly consider monetary factors.

in this paper could be formalized as a series of nonlinear accelerator-multiplier models.[2]

This paper is divided into four sections. The first is a brief review of the attributes of both linear and nonlinear accelerator-multiplier models, which is followed in the second section by an analysis of the behavior of the accelerator model with the quantity of money constant. The third section is an investigation of how the system would behave with the quantity of money varying in a number of different ways. In the last section some implications of the analysis for monetary and fiscal policies are briefly explored.

I. *Formal Attributes of Accelerator-Multiplier Models*

The essential linear accelerator-multiplier model can be written:[3]

$$Y_t = C_t + I_t \tag{1}$$

$$C_t = \alpha Y_{t-1} \tag{2}$$

$$I_t = \beta(Y_{t-1} - Y_{t-2}) \tag{3}$$

where Y = income, C = consumption, I = investment, α = marginal (= average) propensity to consume, β = accelerator coefficient and t is the number of the "day." By substitution, equations (1)-(3) yield:

$$Y_t = (\alpha + \beta) Y_{t-1} - \beta Y_{t-2} \tag{4}$$

Equation (4) is a second-order difference equation; its solution in general is of the form:

$$Y_t = A_1\mu_1{}^t + A_2\mu_2{}^t \tag{5}$$

where A_1 and A_2 depend upon the initial conditions and μ_1 and μ_2 are determined by the values of α and β.

Aside from the effects of the initial conditions, the time series generated by a second-order difference equation can be any one of the following: (1) monotonic equilibrating; (2) cyclical equilibrating; (3) cyclical with constant amplitude; (4) cyclical explosive; (5) monotonic explosive.[4] By itself, no one of these five types of time

[2] Obviously the interest rate and consumer debt affect consumption expenditures also; therefore the consumption coefficient also depends upon the behavior of the monetary system. The "Pigou effect" can be interpreted as a particular relation between the consumption coefficient and the monetary system. Such effects are ignored in this paper.

[3] This stripped model exhibits the characteristics of a linear accelerator-multiplier model which are important for the problems discussed in this paper. The incomes should be interpreted as deviations from a "zero" level of income given by $Y_0 = \lambda/1-\alpha$ where λ could be identified with autonomous investment or "zero income" consumption.

W. J. Baumol, *Economic Dynamics, An Introduction* (New York, 1951), Ch. 10, 11, gives a very simple discussion of the solution to second-order difference equations.

[4] The type of time series generated is determined by the values of μ_1 and μ_2, which in turn depend upon the values of α and β. For a type-1 series, μ_1 and μ_2 are both less than 1, for a type-2, 3, or 4 series μ_1 and μ_2 are conjugated complex numbers, and for a type-5 series μ_1 and μ_2 are both greater than 1.

series is satisfactory for business-cycle analysis. Types 1 and 5 are not cyclical. If they are to be used, either floors or ceilings to income or pushes (systematic or random) from outside have to be posited. A time series of type 2 would in time result in the cycle dying away, so that some systematic or random push is required to maintain the cycle. A time series of type 4 would in time generate fluctuations greater than any preassigned value. Hence floors and ceilings have to be posited to constrain the fluctuations. A type-3 time series is a self-sustaining cycle, but its existence depends upon a particular value of β and, in addition, the time series it generates is "too" regular.

A way out of this difficulty is to have the α and β coefficients vary over the cycle, thus generating a time series which is a combination of the different types of time series. Hicks and Goodwin do this by assuming that the value of β is so great that, unless constrained, an explosive time series is generated, but that constraints, in the form of a maximum depreciation rate and full employment (or the capacity of the capital-goods-producing industries), exist. These constraints force realized investment to be different from induced investment, and, formally, they can be interpreted as changing the value of β. As the value of β is assumed to fall (rise) when income is very high (low) or increasing (decreasing) very rapidly, an acceptably irregular cyclical time series is generated. Obviously by linking explosive, cyclical and damped movements together, any type of time series which is desired can be generated.

A set of formal nonlinear models similar to those of Hicks and Goodwin can be generated by positing that the value of β, the accelerator coefficient, depends upon money-market conditions and the balance sheets of firms. These factors in turn depend upon the relation between the level and rate of change of income and the behavior of the monetary system. In this paper however the mathematical model of the accelerator process will be a simple linear form. It is hoped that what is lost in mathematical neatness may be offset by what is gained in the identifiability of the economics.

So far we have not taken up the effects of the initial conditions. The initial conditions are particularly important in determining the income generated by a type-5 (monotonic explosive) time series for small values of t. To generate a type-5 time series, μ_1 and μ_2 are both greater than 1 in the relation $Y_t = A_1\mu_1{}^t + A_2\mu_2{}^t$. To set off the recursive process two levels of income Y_0 and Y_1 (the initial conditions) are needed, which determine the values of A_1 and A_2. If Y_1 is greater than Y_0 and the ratio of Y_1 to Y_0 is less than μ_2, the smaller root, then A_1, the coefficient of μ_1, the larger root (also called the dominant root), will be negative. As the larger root will in time dominate, a negative

A_1 will in time result in a negative Y_t. Hence if the rate of increase of income given by the initial conditions is less than the smaller root, there will be a turning point in the time series even though the values of α and β are such as to generate a monotonic-explosive time series.[5]

This leads to an alternative way of interpreting the Goodwin-Hicks type of nonlinear accelerator models. When the floors and ceilings become effective, a new set of initial conditions is, in effect, imposed on the time series. If these new "initial conditions" result in the sign of the coefficient of the dominant root changing, then in time the direction of the movement of income will be changed. The effects of monetary constraint can also be interpreted in this manner.

Following Goodwin and Hicks we will assume that the value of β is so large that, unless it is constrained, the accelerator-multiplier process will generate an explosive time series. The solution of the accelerator-multiplier model will be $Y_t = A_1\mu_1^t + A_2\mu_2^t$ where $\mu_1 > \mu_2 > 1$ and the initial conditions are such $(Y_1/Y_0 > \mu_2)$ that A_1 and A_2 are both positive. For the range of magnitudes of Y_1/Y_0 which it seems sensible to posit, A_2 will be much larger than A_1. This means that at the early dates (t small) of the development the weight of μ_2 is high while at the later dates μ_1 dominates. The rate of growth of income generated by the explosive process being considered increases in time, approaching μ_1 as a limit.[6]

The increasing rate of increase of income that such an explosive accelerator process generates will in time be greater than the accepted possible rate of growth of productive capacity. In order to be able to maintain the continuity of the accelerator process, we assume that all the relations are in money terms and that the accelerator process may generate changes in the price level. We will, at a number of points, call attention to some specific effects of price level changes.

II. *The Accelerator Model with the Quantity of Money Constant*

In this and the following section we will derive several time series that result from the interaction of an accelerator-multiplier process

[5] If the two roots are equal, then the solution to the difference equation is $Y_t = A_1\mu_1^t + A_2t\mu_1^t$ (see Baumol, *op. cit.*, Ch. 10, 11). If $Y_1/Y_0 = \mu_1$, then $A_2 = 0$ and a constant-rate-of-growth series is generated. If $Y_1/Y_0 < \mu_1$, then $A_2 < 0$ and in time $Y_t < Y_{t-1}$; if $Y_1/Y_0 > \mu_1$, then $A_2 > 0$ and, at least in the early days, the rate of increase of income is significantly greater than μ_1. In terms of a second-order difference equation, a steady rate of growth of income can be characterized as a knife edge: it requires not only that α and β be such that $\mu_1 = \mu_2 > 1$ but also that $Y_1/Y_0 = \mu_1$ (see S. S. Alexander, "The Accelerator as a Generator of Steady Growth," *Quart. Jour. Econ.*, May 1949, LXIII, 174-97.

[6] In Sections II and III a number of tables will be exhibited to illustrate the results of combining an explosive accelerator-multiplier process with a number of different monetary systems. In each case it is assumed that $\alpha = .8$, $\beta = 4$, $Y_0 = 100$ and $Y_1 = 110$. For these values $\mu_1 = 3.73$, $\mu_2 = 1.07$, $A_1 = 1.1$ and $A_2 = 98.9$ so that $Y_t = 1.1(3.73)^t + 98.9(1.07)^t$. In time Y_{t+1}/Y_t will approach 3.73.

and various types of monetary systems. The monetary systems to be considered are classified in terms of the monetary changes which can take place. Monetary changes are changes in either the velocity of circulation or the quantity of money. Therefore we will consider the following alternative monetary systems: (A) neither velocity nor quantity changes; (B) only velocity changes; (C) only quantity changes; (D) both velocity and quantity change.[7] The first two monetary systems will be considered in this section, the last two in the next section.

Except in the first monetary system, we assume that there exists a fractional reserve banking system. The money supply is changed by either the creation of deposits in exchange for business firms' debts or the destruction of deposits by business firms' repayment of bank debt. That is, the banking system is a commercial banking system rather than one that deals in government and other securities.[8] In all that follows the central bank's relations with the commercial banks are integrated into the "monetary system." For example, an infinitely elastic money supply can be achieved by a central bank lending to commercial banks, or by a central bank purchasing open market paper. Also in a monetary system we include the specialized financial intermediaries.

The income velocity of money and the liquidity preference relation can be characterized as mirror images of each other.[9] When income velocity rises, the liquidity of the economy falls and vice versa. A useful construction is to assume that for each level of money income Y, there exists a minimum quantity of money M_T which is necessary to sustain the volume of payments associated with Y. If M_T is the total quantity of money in existence then there is no money available for portfolio use; we have a maximum income velocity of money V_m for

[7] Cases A and B, where the quantity of money is constant, may be thought of as worlds of 100 per cent money. If at the "initial point" excess liquidity exists, so that velocity can increase, it is Case B, otherwise it is Case A. Case C(1), where the money supply is infinitely elastic, is a world of a paper-money authority which ignores price-level considerations (perhaps a world in which the central bank follows a "needs of business" rule). Case C(2), where the quantity of money has an exogenously determined rate of growth, is a gold-standard world where gold production is autonomous and determines the rate of growth of the money supply. Case D of course is similar to the existing monetary system.

[8] Some of the differences between the classical quantity theory of money and the Keynesian liquidity preference theory of money can be imputed to the way in which the banking system is assumed to operate. The quantity theory approach is consistent with bank lending to business (commercial banking) whereas the liquidity preference theory follows from banks purchasing securities on the open market. In commercial banking an increase in the quantity of money enables a business firm to effect a decision to purchase goods and services. On the other hand, open-market operations substitute money for another asset in the portfolios of the public, and whether or not purchases of goods and services result depends upon the reaction of the public to this change in liquidity.

[9] A. C. Pigou, *Keynes's General Theory* (London, 1951); H. S. Ellis, "Some Fundamentals in the Theory of Velocity," *Quart. Jour. Econ.*, May 1938, LII, 431-72.

each Y, so that $M_T \cdot V_m = Y$. If M is greater than M_T then the actual velocity, V, is less than V_m. The difference between M and M_T is M_L, the amount of money which is held as a liquid asset. If the quantity of money is constant, portfolio money M_L must fall when V rises.

If $V < V_m$ then $M_L > 0$. Abstracting from changes in the quantity of money, with $M_L > 0$, the interest rate is determined by the demand curve for investment, *ex ante* saving, and the terms upon which holders of liquidity are willing to substitute earning assets for money. Similarly, if $M_L = 0$, then the interest rate is determined by the demand for investment, the supply of saving and the terms upon which individuals are willing to hold cash as an asset. With a given money supply in excess of M_T there exists a rate of interest at which households and business firms as a whole are not willing either to increase or to decrease their holdings of money. Any other market interest rate involves either an increase in cash balances so that savings are utilized to increase liquidity, or a decrease in cash balances so that investment is financed from the reservoir of purchasing power. It is assumed that changes in the market rate of interest will affect the amount of investment induced by a given change in income.

Assume that all investment is made by business firms. On a consolidated balance sheet of all firms, investment is represented by an increase in plant, equipment or work in progress, and it will be offset by an increase in liabilities (equity or debt) or a decrease in other assets (cash or liquid assets). Business investment can be equity-financed as a result of either *ex ante* saving by households and firms or a decrease in the cash balances of households. Business investment can be debt-financed as a result of *ex ante* saving by households, a decrease in households' cash balances or by an increase in bank debt of business firms. The financing of investment by a decrease in the cash (liquid assets) balances of firms does not affect either the debt or the equity liabilities of firms: it only makes firms less liquid.

Whereas *ex ante* saving and decreases in the liquidity of households can be used for either debt or equity financing of investment, increases in the quantity of money can be used only for the debt financing of investment. Households, business firms, and banks are sensitive to the composition of the balance sheets of firms; in particular an increase in the ratio of debt to equity or a decrease in the ratio of cash to other assets in firms' balance sheets will make business firms less willing to borrow and households and banks less willing to lend. Hence if investment is financed in such a way as either to increase the ratio of debt to total liabilities or to decrease the liquidity of business firms, the amount of investment induced by a given change in income will fall. The value of the accelerator coefficient therefore depends upon

two variables, the market rate of interest and the structure of the balance sheets of firms. Changes in these variables can dampen what otherwise would be an explosive movement of income.

A. *Neither Velocity nor Quantity Changes*

Using the Swedish concepts,[10] we define $Y_{t-1} - C_t = (1 - \alpha)Y_{t-1}$ as *ex ante* saving. Assuming, as pure accelerator-multiplier models do, that all of investment is induced, then $I_t = \beta(Y_{t-1} - Y_{t-2})$ is identified as *ex ante* investment. From equations (1)-(3), it follows that for $Y_t \geq Y_{t-1}$ it is necessary that $I_t = \beta(Y_{t-1} - Y_{t-2}) \geq (1 - \alpha)Y_{t-1}$, for $Y_t < Y_{t-1}$ it is necessary that $I_t = \beta(Y_{t-1} - Y_{t-2}) < (1 - \alpha)Y_{t-1}$.

With a monetary system in which neither the velocity of circulation nor the quantity of money changes, if *ex ante* investment is greater than *ex ante* saving, the *ex ante* saving has to be rationed among investors, and the market in which this rationing takes place is the money market. The excess of demand over supply results in a rise in interest rates, which will continue until realized investment is equal to *ex ante* saving. In Figure 1, *ex ante* investment is based upon the

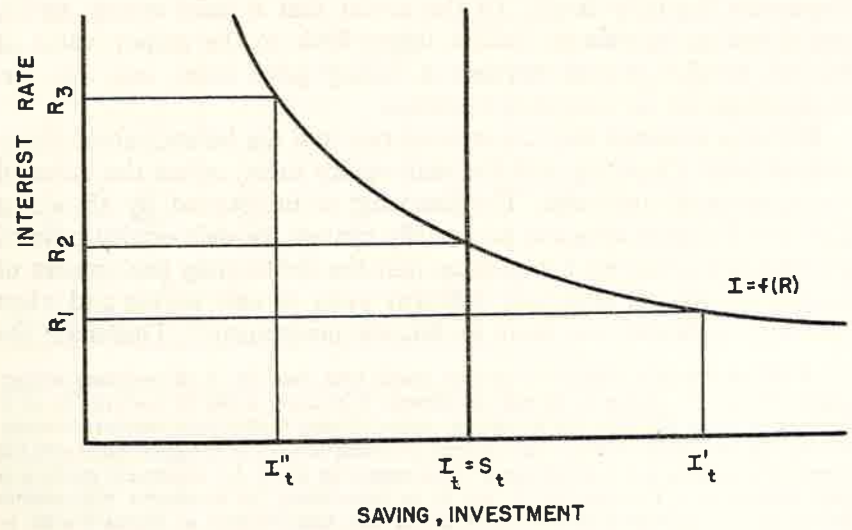

FIGURE 1. RECONCILIATION OF EX ANTE SAVING AND INVESTMENT

rate R_1 so that $\beta(Y_{t-1} - Y_{t-2}) = I_t'$. The inability to finance more than I_t $(=S_t)$ of investment results in a rise in the interest rate to R_2. Such a monetary system leaves no room for an accelerator-multiplier cycle. A necessary condition for the functioning of an accelerator

[10] B. Ohlin, "Some Notes on the Stockholm Theory of Savings and Investment," *Econ. Jour.*, Mar. and June 1937, XLVII, 53-69 and 221-40. Reprinted in American Economic Association, *Readings in Business Cycle Theory* (Philadelphia, 1951), pp. 87-130.

process during an expansion is that a source of financing of investment in addition to *ex ante* saving should exist.[11]

Symmetrically, if *ex ante* saving is greater than *ex ante* investment then an increase in investment is forced so that all of the available financing is absorbed by real investment. If there exists no way in which savings can be utilized other than in investment, then the terms upon which firms can finance investment must change so that realized investment is greater than *ex ante* investment. This equality of *ex ante* saving and realized investment stabilizes income, thereby halting the "inducement to disinvest."

B. *Only Velocity Changes*

With a constant money supply, realized investment can differ from *ex ante* saving only if the velocity of circulation of money changes. We will first take up the purely mechanical implications of the existence of a floor and a ceiling to velocity. We will then consider the effects on the value of the accelerator coefficient of changes in velocity when no excess liquidity exists and when excess liquidity exists (the Keynesian liquidity trap). To the extent that a fixed money supply and a ceiling to velocity set an upper limit to the money value of income, secular growth requires a falling price level, and this has implications for the accelerator process.

We have assumed that the interest rate and the balance-sheet structure of firms (liquidity and the debt-equity ratio) affect the value of the accelerator coefficient. The financing of investment by absorbing idle cash balances does not necessarily change the debt-equity ratio of business firms, for we can assume that the debt-equity preferences of households are not strikingly different when *ex ante* saving and when idle cash balances are used to finance investment.[12] Therefore the

[11] A fall in the price level of investment goods may result in S_t of monetary savings being sufficient to finance I_t of real investment. Conversely a rise in the price level of investment goods will lower the amount of real investment that a given amount of money savings can finance. In Figure 1 the savings curve can be read as a supply curve and the investment curve as the demand curve (with respect to price) for investment goods at a fixed interest rate. Then reading R_2 and R_1 as price levels, the accelerator phenomenon determines the price level of investment goods. This interpretation of Figure 1 must be what a writer who uses a ceiling to investment-goods production in his models has in mind (for example, Goodwin, *op. cit.*). In the original interpretation of Figure 1, even if I_t' of investment is financed, the supply conditions of investment goods (with respect to price) may be such that spending I_t' on investment goods results in a rise in the price of investment goods; as indicated earlier the accelerator process can lead to a rising price level.

[12] J. G. Gurley and E. S. Shaw, "Financial Aspects of Economic Development," *Am. Econ. Rev.*, Sept. 1955, XLV, 515-38, discuss the effect of available assets on saving behavior. It may be true that the asset preferences of households when using cash balances are different from their preferences when using *ex ante* saving to finance firms. In this connection, the legal and traditional limitations on the portfolios of financial intermediaries no doubt tend to affect business investment.

balance sheets of investing firms do not deteriorate during an expansion financed by increasing velocity. Of course the liquidity of households and firms is reduced but, unless the liquidity trap is operative, this is reflected in the interest rate. Therefore in this section only the interest rate and, in the liquidity-trap situation, the changes in liquidity at a constant interest rate can affect the accelerator coefficient.

Assume that a cumulative rise in income is set off. This increases the quantity of money needed for transaction purposes and, therefore, as the process continues there are progressively smaller asset holdings of money which can be used to finance investment in excess of *ex ante* saving. The highest attainable level of money income is that level at which all of the available money supply is required for transactions (see Table I). At that income realized investment cannot exceed *ex ante* saving. Realized investment equal to *ex ante* saving results in a constant income which, given the accelerator assumption, induces zero investment. Ignoring any effects that the interest-rate and balance-sheet changes accompanying velocity increases have upon the accelerator coefficient, a monetary system with a constant quantity of money may impose a ceiling to money income. This ceiling is not determined by full employment or by the capacity of the investment goods industries; it is determined by the limited ability of changes in velocity to finance investment.

Table I.—Only Velocity Changes

(Constant Money Supply—No Interest-Rate Effects)

Time	Accelerator Process $\alpha=.8, \beta=4.0$ $Y_0=100, Y_1=110$					Monetary System Money Supply=100 Maximum Velocity=2	
	Y	C	Savings *Ex Ante*	Investment *Ex Ante*	Investment Realized	Investment Financed by ΔV[a]	Realized Velocity
0	100	—	—	—	—	—	1.00
1	110	80	20	—	30	10	1.10
2	128	88	22	40	40	18	1.28
3	174	102	26	72	72	46	1.74
4	200	139	35	184	61	26	2.00
5	200	160	40	104	40	0	2.00
6	160	160	40	0	0	−40	1.60

[a] Investment in excess of *ex ante* saving. Obviously negative investment financed by ΔV means that *ex ante* saving is greater than investment.

Symmetrically if a minimum velocity exists, a floor to money income exists. However the floor is not entirely symmetrical with the ceiling, and in this paper the lower turning point is essentially unexplained.

Let us examine what would be happening in the money market during a process such as is detailed in Table I. Ignoring the liquidity trap, a rise in transaction money as income rises means that with a constant money supply portfolio money becomes scarcer. The interest rate at which cash can be withdrawn from portfolios into the income stream rises as asset money is used to finance investment in excess of saving. With a fixed quantity of money and a rise in income, the balance sheets of households and firms show a smaller ratio of asset cash to total assets, liquidity decreases. The decrease in liquidity and the rise in the interest rate both tend to decrease the accelerator coefficient.

Alternatively, on the downswing *ex ante* investment is smaller than *ex ante* saving. With a constant money supply, this excess saving is

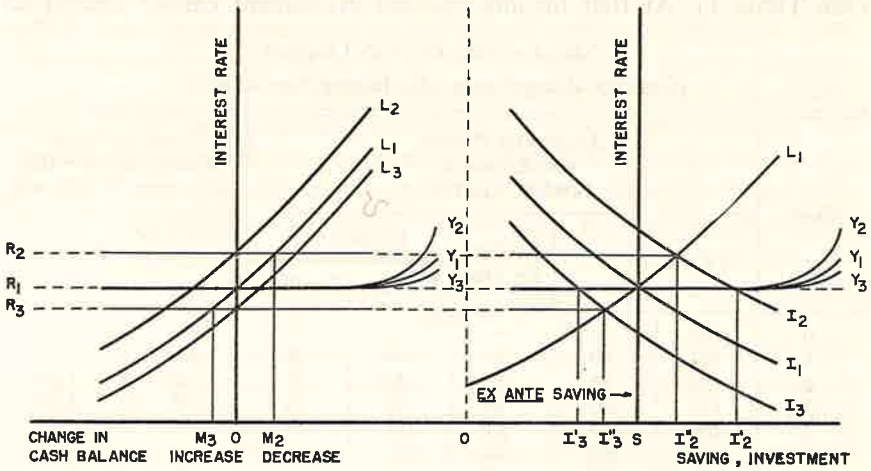

FIGURE 2. SAVING, INVESTMENT, AND CASH BALANCES

absorbed by a reduction in velocity. Money available for asset purposes increases as it is withdrawn from the income stream. The interest rate falls and the liquidity of the community rises so that the amount of disinvestment induced by the given downward shift in demand decreases. Both on the upswing and the downswing, the monetary system which is based solely upon changes in velocity acts as a stabilizer of realized induced investment unless the fall in income is so great that the money released from transaction purposes lowers the interest rate to the floor interest rate of the liquidity trap. At this interest rate the stabilizing effect upon aggregate disinvestment of the fall in financing terms will cease, although increasing liquidity can continue to act as a stabilizer.[13]

Figure 2 illustrates the use of cash balances to finance investment

[13] Increasing liquidity raising the consumption coefficient is of course the "Pigou effect."

and to offset *ex ante* saving. At the interest rate R_1, and income Y_0, the velocity of circulation of money remains constant. This is illustrated by the L_1 curve intersecting the zero change in cash balances line at R_1. At higher interest rates cash assets would be freed to finance investment; at lower interest rates saving would be absorbed by cash balances. The amount of investment which can be financed at any interest rate is equal to the sum of *ex ante* saving and the change in cash balances. Assume that income rises so that at the interest rate R_1, I_2' of investment is induced. The I_2 curve illustrates how the value of the accelerator would be changed by a change in interest rates. The excess of demand over the supply of finance results in a rise of the interest rate to R_2. As I_2'' is greater than *ex ante* saving, income will rise and the transaction demand for cash will increase. This will raise the schedule relating the change in cash balances to the interest rate to L_2, so that the interest rate at which investment will be financed by a fall in liquidity will be higher.

If a fall in income shifts the investment demand curve to I_3, *ex ante* investment is I_3'. With a constant money supply the excess of *ex ante* saving over induced investment will depress the interest rate, and realized investment will be $I_3'' > I_3'$, OM_3 being added to cash balances. As $S > I_3''$ income will fall, and this will shift the liquidity curve downward so that cash balances can be used to finance investment at an interest rate lower than R_1.

If the cash balance-interest rate relation is as the Y_3, Y_2 and Y_1 set of curves indicate, then excess liquidity exists; this is the Keynesian liquidity trap situation. With an investment curve I_2, $I_2' - S$ of investment will be financed by a decrease in cash balances; and if the investment curve is I_3, $S - I_3'$ will be added to cash balances. In both cases no change in interest rates will occur. In the Keynesian liquidity trap situation the money market damps down neither the "boom" nor the "bust." On the boom side, the liquidity trap will exist until the need of cash for transactions absorbs a sufficiently large portion of the money supply so that the Keynesian liquidity trap comes to an end. There is no endogenous limiting factor to the liquidity trap on the downswing aside from the effect that improved liquidity has upon firms' balance sheets. Therefore the Keynesian liquidity trap situation allows full scope to an explosive accelerator coefficient. And in the upswing, an explosive accelerator process will generate greater increases in money demand than the increases in productive capacity, so that a strong accelerator in combination with excess liquidity will generate large price increases.

Either the ceiling to velocity or the effect of rising interest rate and decline in liquidity upon the accelerator coefficient will break th cumulative expansion. A fall in money income will occur. The quantity of money needed for transactions falls, and *ex ante* saving which is not realized in investment will result in the addition of money to portfolios. If the price level does not fall during a depression the ceiling real income remains fixed, while if the price level falls, even though the ceiling money income remains fixed, the ceiling real income rises.

Net investment implies an increase in productive capacity. With a constant money supply and in effect a ceiling to velocity, larger real incomes can be realized only if the price level falls. To the extent that the accelerator inducement to invest is large only when income is approximately equal to productive capacity, strong expansions can only occur if the price level falls secularly.

The effect of the expectation that in the long run the price level will fall is to increase the expected pay-off period of an investment. This is equivalent, in its effect upon investment by firms, to a rise in interest rates with a constant price level, so that a falling price level will tend to lower the value of the accelerator coefficient. Therefore the business cycle will be characterized by weaker booms than would occur with a permissive monetary system. Such a monetary system will be associated with a tendency toward relatively stable income for, unless liquidity is greatly increased during a downswing, long periods in which realized investment exceeds *ex ante* saving cannot occur.

III. *The Accelerator Model with Quantity of Money Variable*

In this section we will consider two monetary systems, those in which only the quantity of money can change and those in which both the quantity of money and its velocity can change.

We assume that commercial banks create money by lending to business firms. The maximum realized increase in the money supply is equal to the difference between *ex ante* investment and *ex ante* saving:

$$\Delta M = ex\ ante\ I - ex\ ante\ S = \Delta Y$$

Assume that $V = \dfrac{Y}{M} = \dfrac{\Delta Y}{\Delta M} = 1$. The increase in the money supply in the hands of households is the asset which makes the change in net worth equal to *ex ante* investment.[14] As income velocity is 1, there will

[14] Assume that *ex ante* $I >$ *ex ante* S, realized $I =$ *ex ante* I; also that (*ex ante* I − *ex ante* S) is financed by an increase in bank debt. The changes in the consolidated balance sheets of households, business firms and banks will be:

Households

Debt and Equity of Firms	+(*ex ante* S)	Net Worth	+(*ex ante* I)
Demand Deposits	+(*ex ante* I − *ex ante* S)		

be no net change in the quantity of money that individuals hold as assets. This is equivalent to assuming that the interest rate at which banks lend to business is the interest rate at which money and earning assets are substituted in household portfolios.[15] The only relevant monetary change in these models is in the quantity of money.

When the money supply increases at an independently given rate, the autonomous increase in the money supply is not necessarily equal to the difference between *ex ante* investment and *ex ante* saving. If the increase in the money supply is greater than the difference between *ex ante* investment and *ex ante* saving we assume that this difference accumulates in the banking system (as excess reserves) and can be used to finance future investment. If the increase in the money supply is less than the difference between *ex ante* investment and *ex ante* saving, realized investment will be less than *ex ante* investment and the increase in income will be equal to the increase in the money supply.

For each monetary system we will first investigate the mechanical properties of these relations, assuming that the accelerator coefficient does not change, and then investigate the possible effects of the associated money market and financing developments upon the value of the accelerator coefficient.

A. *Quantity Changes but Not Velocity*

Two monetary systems in which only the quantity of money can change will be taken up. In the first, the money supply will be assumed to be infinitely elastic, and in the second the money supply will be assumed to increase at a fixed arithmetic or geometric rate.

1. *Infinitely elastic money supply*. If the quantity of money can increase without limit then no matter what the difference between *ex ante* investment and *ex ante* saving, the difference can be financed. Also we can assume that the terms upon which the banking system

Firms

Productive Assets	+(*ex ante* I)	Debt and Equity to Households	+(*ex ante* S)
Demand Deposits	(no change)	Debts to Banks	+(*ex ante* I − *ex ante* S)

Banks

Debts of Firms	+(*ex ante* I − *ex ante* S)	Demand Deposits	+(*ex ante* I − *ex ante* S)

[15] Alternatively if the liquidity-trap rate of interest rules, even if $V > 1$, the rise in the quantity of money in excess of transaction needs can all be absorbed by households' portfolios without lowering the interest rate. However, in this case any rise (virtual) in the interest rate would imply a substitution of earning assets for money in the portfolios of households. This then becomes a case of financing investment from cash balances. If $V > 1$ the money supply and firms' debts to banks do not increase as rapidly as income.

lends do not change. Such a monetary system is consistent with the existence of an explosive accelerator process since it permits a cumulative rise in money income. Is there anything inherent in the operations of such a monetary system which will lead to a dampening of the accelerator process? (We will ignore the political repercussions of the cumulative rise in prices which is implicit in a full-employment situation in which the rate of growth of money income is greater than that of productive capacity.)

TABLE II.—INFINITELY ELASTIC MONEY SUPPLY

(Constant Velocity—No Interest-Rate Effects)

Time	Accelerator Process $\alpha=.8$ $\beta=4$ $Y_0=100$					Monetary System All *ex ante S* used for equity financing. All increases in money used for debt financing.	
			Savings	Investment			
	Y	C	*Ex Ante*	*Ex Ante* $\beta(Y_{t-1}-Y_{t-2})$	Realized	Δ Money Supply	ΔEquity Financing / ΔTotal Investment
0	100.	—	—	—	—	—	—
1	110.	80.	20.	—	30.	10.	.67
2	128.	88.	22.	40.	40.	18.	.55
3	174.	102.	26.	72.	72.	46.	.36
4	323.	139.	35.	184.	184.	149.	.19

During an expansion, the increase in money supply occurs as investing firms add bank debt to their liabilities (see Table II). Assuming that the percentage distribution of *ex ante* saving between debt and equities of business firms is constant, a cumulative explosive expansion on the basis of the creation of money will (*ceteris paribus*) result in a fall in the ratio of equity to debt in the balance sheet of firms.[16] Even if the terms upon which firms can borrow are unchanged by the

[16] Total induced investment is $\beta(Y_t - Y_{t-1})$. *Ex ante* saving is equal to $(1-\alpha)Y_t$. Assuming that a constant proportion of *ex ante* saving is used for equity financing, the latter is $\lambda(1-\alpha)Y_t$. The ratio of the change in equity to total investment, therefore is:

$$\frac{\lambda(1-\alpha)Y_t}{\beta(Y_t - Y_{t-1})} = \frac{\lambda(1-\alpha)}{\beta\left(1 - \frac{Y_{t-1}}{Y_t}\right)}$$

The general solution to the second-order explosive accelerator process is of the form $Y_t = A_1\mu_1^t + A_2\mu_2^t$ where $\mu_1 > \mu_2 > 1$. Therefore, we can write:

$$\frac{Y_{t-1}}{Y_t} = \frac{A_1\mu_1^{t-1} + A_2\mu_2^{t-1}}{A_1\mu_1^t + A_2\mu_{2t}} = \frac{1 + \frac{A_2}{A_1}\left(\frac{\mu_2}{\mu_1}\right)^{t-1}}{\mu_1 + \left(\frac{A_2}{A_1}\right)\left(\frac{\mu_2}{\mu_1}\right)^{t-1}\mu_2}$$

deterioration of their balance sheets, borrowers' risk will rise.[17] This will lower the amount of investment induced by a given rise in income. Hence, even with a monetary system that permits all of *ex ante* investment to be realized, the financing of investment by bank debt can result in lowering the accelerator coefficient which in turn lowers the rate of increase of income. This continues until the accelerator coefficient falls sufficiently to replace the explosive by a cyclical time series, in which there eventually occurs a fall in income. With a fall in income, the excess of *ex ante* saving over induced investment will be utilized to reduce bank debt. Also, the failure of some firms which have relied heavily upon debt financing will result in the substitution of equity for debt in balance sheets. Both changes during the downswing raise the ratio of equity to debt in firms' balance sheets[18] which acts as a

The limit of $\left(\frac{\mu_2}{\mu_1}\right)^t_{t\to\infty} = 0$, therefore the limit of $\left(\frac{Y_{t-1}}{Y_t}\right)_{t\to\infty}$ is $\frac{1}{\mu_1}$.

Hence $\frac{\lambda(1-\alpha)Y_t}{\beta(Y_t - Y_{t-1})}$ approaches as a limit $\frac{\lambda(1-\alpha)}{\beta\left(1-\frac{1}{\mu_1}\right)}$.

In the early stages of an explosive accelerator process the ratio of $\frac{Y_{t-1}}{Y_t} > \frac{1}{\mu_1}$. Therefore, the ratio of equity financing to total investment decreases as the accelerator process continues.

[17] M. Kalecki, "The Principle of Increasing Risk," *Economica*, N. S., Nov. 1937, IV, 440-47.

[18] On the downswing (*ex ante* $S >$ *ex ante* I), the balance sheets of the three sectors change as follows:

Banks

Business Debt	$-$(*ex ante S* $-$*ex ante I*) $= -\Delta M$	Demand Deposits	$-$(*ex ante S* $-$*ex ante I*) $= -\Delta M$

Firms

Capital Equipment	$+$*ex ante I*	Debt and Equities to Households	$+$*ex ante S*
		Debt to Banks	$-$(*ex ante S* $-$*ex ante I*) $= -\Delta M$

Households

Demand Deposits	$-$(*ex ante S* $-$*ex ante I*) $= -\Delta M$	Net Worth	$+$*ex ante I*
Business Assets	$+$*ex ante S*		

If failures occur in the account of households labeled Business Assets, equities will be substituted for debt and in the account of business firms labeled Debt and Equities to Households, equity will be substituted for debt. Also as business firms fail banks acquire titles and debts which are considered unsuitable for bank portfolios. The sale of such assets to the public results in the substitution of business assets for demand deposits in stabilizer. The endogenous limits to an explosive accelerator process, in the absence of restrictions on the money supply, are the deterioration of firms' balance sheets due to debt-financing of investment on the upswing; and the reverse circumstances during the liquidation process on the downswing.

Two possible offsetting factors to the increasing debt-equity ratio in the financing of investment during an explosive expansion are an increase in the ratio of *ex ante* saving flowing to equities and the capital gains that accompany an increase in the price level of capital goods. As *ex ante* saving finances a decreasing proportion of total investment during an explosive expansion, a possible increase in the proportion of *ex ante* saving flowing to equities cannot for long prevent a deterioration of the balance sheets of firms. If, however, cumulative price-level inflation is politically permissible a deterioration of firms' balance sheets need not occur. Business firms are borrowers and the real burden of a debt decreases with a rise in the price level. If the assets of business firms are valued at their current replacement costs, then the rising price level raises the equity account. Such capital gains improve the balance sheets of firms and they occur generally in an inflation. The price-level rise plus the flow of *ex ante* saving to equity investment may be sufficient to keep the debt-equity ratio constant, thereby preventing any deterioration in the balance sheets of firms. However, this requires an increasing rate of change in the price level of capital goods.[19] Nevertheless, if an explosive inflation is politically

the public portfolios, and in a net reduction of demand deposits. These changes obviously do not affect the net worth of households and the capital equipment accounts. However, as the value of productive capacity may be reduced during a downturn, the value of the capital equipment account of firms and the net worth account of households may be reduced; the equity liabilities of firm and equity assets of households lose a part or all of their value. This in turn can affect the "subjective" preferences of households and firms so that liquidity preference rises.

[19] In the arithmetic example of Table II, in time-period 3, only .36 of the total new investment was financed by savings. If, in period 3, the price level of capital goods rose so that the value of existing capital goods rose by 2.0, then the ratio of the increase in equity to the increase in assets would be .5. In period 4 only .19 of a larger total investment was financed by savings. For the ratio of the increase in equity to the increase in the value of the assets to be .5, the value of existing capital must rise by 11.4. As total assets in period 4 are presumably only slightly larger than in period 3, this implies that the rate of increase in the price level of capital goods must rise if a constant ratio of equity to total assets is to be maintained. For example:

Period	3	4
Saving, *ex ante*	26.0	35.0
I realized	72.0	184.0
Δ money	46.0	149.0
Required Δ value of existing capital	20.0	114.0
Δ equity $=S+\Delta$ value	46.0	149.0
Δ assets $=I$ realized $+\Delta$ value	92.0	298.0
Ratio of Δ equity to Δ assets	.5	.5

tolerable, there is no endogenous reason why an accelerator process with an infinitely elastic money supply need come to a halt.

Therefore, at least two monetary situations allow full scope to an explosive accelerator process: the Keynesian liquidity trap and an infinitely elastic money supply. It is perhaps no accident that the emphasis upon "real" floors and ceilings as causes of the nonlinearity of the accelerator coefficient occurred at a time when the high volume of government bonds outstanding and their support by central banks made the money supply in fact infinitely elastic. An era of tight money on the other hand naturally leads to an examination of the monetary prerequisites for the operation of the accelerator phenomena.

2. *Money supply increases at a fixed rate.* A monetary system in which the rate of growth of the money supply is exogenously given, for example a fractional reserve banking system based upon a gold standard, is equivalent to an infinitely elastic money supply if the difference between *ex ante* investment and *ex ante* saving does not exceed the per-period growth of the money supply. The only endogenous limitation to expansion in this case comes from the deteriorating balance sheets and liquidity of business firms, as is true with an infinitely elastic money supply. The interesting alternative exists when the difference between induced investment and *ex ante* saving is greater than the rate of growth of the lending ability of banks.

Throughout this section we will assume that at the initial period the banking system does not possess excess liquidity. Hence the available financing is equal to *ex ante* saving plus the possible increase in the money supply. If induced investment is equal to or greater than this, realized investment will be constrained to the available financing. In this case income will grow at the same rate as the money supply.[20]

(a) *Arithmetic rate of increase in the money supply.* If the money supply increases by a fixed amount per period (constant arithmetic rate of increase), income will grow at this rate until *ex ante* saving increases sufficiently so that induced investment per period becomes less than the available financing. When this happens, the per-period increase in income will fall below what it had been, and therefore induced investment will decrease. The downturn occurs when *ex ante* saving catches up with the expansion process so that all of the investment induced by the constant arithmetic rate of growth of income can be realized without using all of the newly available credit.[21] (This case is illustrated in Table III.)

[20] $\beta(Y_t - Y_{t-1}) > (1-\alpha)\, Y_t + \Delta M$ and $Y_t = M_t$; so that $Y_{t+1} = \alpha Y_t + (1-\alpha) Y_t + \Delta M$; $Y_{t+1} = Y_t + \Delta M$.

[21] In an accelerator-multiplier model a necessary condition for $Y_t > Y_{t-1}$ is that $\beta(Y_{t-1} - Y_{t-2}) > (1-\alpha) Y_{t-1}$. We posit an arithmetical increase in the money supply per period of ΔM so

TABLE III.—ARITHMETICALLY INCREASING MONEY SUPPLY
(Constant Velocity—No Interest-Rate Effects)

Time	Accelerator Process $\alpha=.8$ $\beta=4$					Monetary System +10 per time period
	Y	C	Savings *Ex Ante*	Investment Induced $\beta(Y_{t-1}-Y_{t-2})$	Investment Realized	Investment Financed by Increased Money Supply
0	100.0	—	—	—	—	—
1	110.0	80.0	20.0	—	30.	+10.0
2	120.0	88.0	22.0	40	32.	+10.0
3	130.0	96.0	24.0	40	34.	+10.0
4	140.0	104.0	26.0	40	36.	+10.0
5	150.0	112.0	28.0	40	38.	+10.0
6	160.0	120.0	30.0	40	40.	+10.0
7	168.0	128.0	32.0	40	40.	+ 8.0
8	166.4	134.4	33.6	32	32.	− 1.6[a]

[a] In time period 7, *ex ante* $S+\Delta M>$ *ex ante* I; therefore $Y_7-Y_6<\Delta M$. As a result, in time period 8 the accelerator expansion is broken.

During the expansion, the demand for financing is always greater than the available supply; the money market constrains investment. When the arithmetic increase in income becomes less than the increase in the money supply financing conditions ease. The resulting decline in the rate of interest may act to increase the inducement to invest (decrease the inducement to disinvest); this possibility is ignored in Table III. Since the banking system finances a decreasing proportion of realized investment during the expansion, the deterioration of the balance sheets of investing firms will be limited during such an expansion.

When income declines, the autonomous increases in the money supply result in an accumulation of excess reserves in the banking system, and *ex ante* saving in excess of induced investment results in a repayment of bank debt by firms. These changes should brake the decline in income.

The accumulation of excess reserves by banks and the improved balance sheets of firms during the downswing implies that if an expansion begins it will not at once be constrained by the money-market and balance-sheet effects. If the arithmetic rate of growth of the money supply is small compared to the accumulation of financing ability during the decline in income, a sharp fall in investment will occur at the date that the accumulated financing ability is absorbed, thereby decreasing the per-period increase in income. The smaller increase in income will lead to a fall in induced investment, and a sharp fall in income may occur. A constant arithmetic rate of increase of the money supply in conjunction with an explosive accelerator process will tend to generate a cyclical time series.

that the available financing is $(1-\alpha)Y_{t-1}+\Delta M$: hence if $\beta(Y_{t-1}-Y_{t-2})\geqq(1-\alpha)Y_{t-1}+\Delta M$ then realized investment is $(1-\alpha)Y_{t-1}+\Delta M$. Hence $Y_t=Y_{t-1}+\Delta M$ so that $\beta(Y_t-Y_{t+1})=\beta\Delta M$ which we once again assume $>(1-\alpha)(Y_{t-1}+\Delta M)$ so that $Y_{t+1}=Y_{t-1}+2\Delta M$. Eventually $\beta(Y_{t+n}-Y_{t+n-1})=\beta\Delta M<(1-\alpha)(Y_{t-1}+n\Delta M)+\Delta M$; so that $Y_{t+n+1}<Y_{t+n}+\Delta M$; therefore $\beta(Y_{t+n+1}-Y_{t+n})<\beta(Y_{t+n}-Y_{t+n-1})$ and the accelerator process turns down.

(b) *Geometric rate of increase in the money supply.* Consider a money supply that increases at a constant geometric rate, μ_3. As was noted earlier the solution of an explosive accelerator process can be written as $Y_t = A_1\mu_1^t + A_2\mu_2^t$ with $\mu_1 > \mu_2 > 1$ with A_1 and A_2 depending upon the initial conditions. That is, the rate of growth of income is a weighted average of the two rates of growth μ_1 and μ_2. If μ_3, the rate of growth of the money supply, is greater than (or equal to) μ_1, the greatest rate of growth that income can achieve, the system behaves as if the money supply were infinitely elastic. Hence the cases that have to be examined are when $\mu_1 > \mu_2 > \mu_3 > 1$ and when $\mu_1 > \mu_3 > \mu_2 > 1$.

Take first the case in which $\mu_1 > \mu_2 > \mu_3 > 1$. With no excess liquidity, the maximum attainable rate of growth of income is the rate of growth of the money supply. To sustain this rate of growth, it is necessary that induced investment be equal to or greater than the available financing. When the rate of growth of the money supply, and therefore the rate of growth of income, is less than μ_2 induced investment will not be large enough to absorb the available financing.[22] The rate of growth of income will be smaller than the rate of growth of the money supply, and this new smaller rate of growth of income also will not be sustained. These progressively smaller rates of growth of income

[22] Assume $M_{t-1}=Y_{t-1}$ and $M_t=Y_t=\mu_3M_{t-1}=\mu_3Y_{t-1}$.

$$\beta(\mu_3-1)Y_{t-1}-[(1-\alpha)\mu_3Y_{t-1}+(\mu_3-1)\mu_3M_{t-1}]\geqq 0$$

is necessary for $Y_{t+1}=\mu_3Y_t$. Therefore $\beta(\mu_3-1)-(1-\alpha)\mu_3-(\mu_3-1)\mu_3-\epsilon=0$, so that $\mu_3^2-(\alpha+\beta)\mu_3+\beta+\epsilon=0$. It follows that

$$\mu_3=\frac{\alpha+\beta\pm\sqrt{(\alpha+\beta)^2-4(\beta+\epsilon)}}{2}.$$

The relevant root is

$$\mu_3=\frac{\alpha+\beta-\sqrt{(\alpha+\beta)^2-4(\beta+\epsilon)}}{2}$$

and if $\epsilon=0$ (induced investment is equal to *ex ante* saving plus the increase in the money supply), $\mu_3=\mu_2$; if $\epsilon>0$ (induced investment greater than *ex ante* saving plus the increase in the money supply) $\mu_3>\mu_2$. Therefore a rate of growth of the money supply equal to or greater than the smaller root of the accelerator process is a necessary condition for self-sustained growth.

will in time result in insufficient induced investment to offset *ex ante* saving and at this date income will fall. Therefore, if the rate of growth of the money supply is smaller than the smallest rate of growth that the accelerator process, if unconstrained, would generate, an upper turning point in income will be produced.[23]

The argument as to what happens once income turns down for a geometric rate of increase in the money supply is essentially the same as for an arithmetic increase in the money supply. Excess reserves accumulate in the banking system and firms' balance sheets improve during the downward movement. Once a sufficient upward movement again begins, an unconstrained expansion can take place until the excess liquidity is absorbed, at which time the rate of growth of the money supply will again constrain the rate of growth of income. A money supply growing at "too small" a rate will lead to a cyclical rather than a steady-growth time series.

If the rate of growth of the money supply is equal to the smaller root of the accelerator process (*i.e.*, $\mu_3 = \mu_2$), both income and the money supply will grow at this rate. Throughout this process the ratio of *ex ante* saving to bank financing of investment will be constant. If this ratio is consistent with the balance-sheet goals, there is nothing in this process which would lead to a downturn in income. Also this rate of growth of income may be consistent with a fairly stable price level. Steady growth may result from combining an explosive accelerator process and an appropriately increasing money supply.[24]

Consider now the second case, in which $\mu_1 > \mu_3 > \mu_2 > 1$. In this case the rate of growth of income during any time period will depend upon the weight of the two roots. If the weight of μ_2 is high, then the accelerator process will generate a rate of growth of income less than the rate of growth of the money supply. However, since $\mu_1 > \mu_2$, in time μ_1 will dominate the rate of growth of income so that income will be increasing faster than the money supply. The money supply does not constrain the growth of income until the total growth of income equals the total growth of the money supply. Whether this case results in steady growth or in a downturn of income depends upon what happens to the accelerator coefficient once the monetary constraint becomes effective.

[23] This can be demonstrated by noting that $Y_0 = A_1 + A_2$ and $Y_1 = A_1\mu_1 + A_2\mu_2$ and given that $\mu_1 > \mu_2 > \mu_3 > 0$ and $Y_1 = \mu_3 Y_0$ then $A_1 = Y_0 - A_2$; $\mu_3 Y_0 = (Y_0 - A_2)\mu_1 + A_2\mu_2$ so that

$$\frac{Y_0(\mu_3 - \mu_1)}{\mu_2 - \mu_1} = A_2.$$

As $Y_0 > 0$, $\mu_3 - \mu_1 < 0$ and $\mu_2 - \mu_1 < 0$, $A_2 > 0$.

Also $A_2 = Y_0 - A_1$, $\mu_3 Y_0 = A_1\mu_1 + (Y_0 - A_1)\mu_2$ so that

$$\frac{Y_0(\mu_3 - \mu_2)}{\mu_1 - \mu_2} = A_1.$$

As $Y_0 > 0$, $\mu_3 - \mu_2 < 0$ and $\mu_1 - \mu_2 > 0$, $A_1 < 0$.

A_1 the coefficient of the dominant root μ_1 is negative. As $A_1\mu_1 + A_2\mu_2 > A_1 + A_2$ and $\mu_1 > \mu_2$ it follows that $|A_2| > |A_1|$. However in time $A_1\mu_1^t + A_2\mu_2^t$ will be <0, so income must turn down.

[24] That is, the Harrod-Domar case of steady growth can be the result of appropriate monetary conditions.

At the beginning of such an explosive expansion the rate of growth of income is less than the rate of growth of the money supply. At the date when the total growth of income becomes equal to the total growth of the money supply the rate of growth of income will be greater than the rate of growth of the money supply. Therefore at some intermediate date, the rate of growth of income will be the same as the rate of growth of the money supply. This rate of growth of income will induce sufficient investment, at the financing terms and balance sheets ruling, for the rate of growth of income to increase. Therefore if the rate of growth of income is constrained to the rate of growth of the money supply, and the accelerator coefficient does not change, a sufficient amount of investment will be induced to generate a rate of growth of income greater than the rate of growth of the money supply.

However until the increase in income and in the money supply becomes equal, this system operates with excess liquidity. At the date that the excess liquidity is absorbed, the rate of growth of income will be greater than the rate of growth of the money supply so that when the monetary constraint becomes effective two things will occur: the rate of growth of income will fall and financing terms will rise. When financing terms were relatively easy because of excess liquidity a rate of growth of income equal to the rate of growth of the money supply induced sufficient investment to increase the rate of growth of income. However in a suddenly tight money market financing terms may so change that the accelerator coefficient will fall, and this can lead to a fall in income.

Nevertheless, if the money supply is growing at a geometric rate greater than the smaller root of the accelerator process, a constant rate of growth of income may be generated. In this case money income will grow at a faster rate than if the money supply grew at the rate given by the smaller root. Hence such a steady rate of growth of income can be associated with a substantial rate of increase in the price level. In addition, the ratio of bank financing to *ex ante* saving increases as the rate of growth of the money supply increases.

If the accelerator falls as a result of the tightening of the money market, income can turn down. The behavior of the economy with this monetary system on the downturn and on subsequent expansions would be essentially the same as in the previous case where the rate of growth of the money supply was smaller than the smaller root of the accelerator process.

B. *Both Velocity and Quantity Change*

The earlier consideration of the interaction of an otherwise explosive accelerator-multiplier process with monetary systems in which only changes in velocity and changes in the quantity of money can occur enables us to consider monetary systems in which both quantity and velocity of money can change. We first assume that the quantity of money is changing but that velocity is greater than 1, we then consider the effects of changing velocity. Finally we take up changes in liquidity preference.

1. In the cases where investment in excess of *ex ante* saving is financed by an increase in the quantity of money, we assumed that the income velocity of money was 1. We can now drop this assumption. If income velocity is greater than 1, and if an excess of *ex ante* investment over *ex ante* saving is financed by an increase in the quantity of money, then excess liquidity results. This excess liquidity can be utilized to finance investment.

Assume that the excess liquidity resulting from an investment initially financed by the banks is used to substitute business debt or equities to the public for business debt to banks. If $\Delta M = Y_t - Y_{t-1}$ and $V > 1$, then new transaction cash is

$$\frac{\Delta M}{V}, \text{ and asset cash is } \Delta M - \frac{\Delta M}{V} = \left(1 - \frac{1}{V}\right)\Delta M.$$

After the public purchases business debts or equities, the net increase in debt to banks is

$$\frac{1}{V}(Y_t - Y_{t-1})$$

and investment is $Y_t - \alpha Y_{t-1}$, therefore:

$$\frac{\Delta\text{Bank Debt}}{\Delta\text{Total Assets}} = \frac{\dfrac{Y_t - Y_{t-1}}{V}}{Y_t - \alpha Y_{t-1}} = \frac{1}{V}\frac{Y_t - Y_{t-1}}{Y_t - \alpha Y_{t-1}}$$

As an explosive accelerator process takes hold, the ratio $\dfrac{Y_t - Y_{t-1}}{Y_t - \alpha Y_{t-1}}$ rises and the ratio of the change in bank debt to the change in total assets approaches $\dfrac{1}{V}$. If the public's distribution of *ex ante* saving and excess liquidity between debt and equity assets is constant during an expansion, the balance sheets of business firms deteriorate. As the weight of bank financing is smaller than in the case of unit velocity, the deterioration will not be so rapid as in the case in which bank creation of money is the sole technique by which investment in excess of *ex ante* saving can be financed. Therefore, the possibility that the deterioration of firms' balance sheets will lower the accelerator coefficient is smaller.

2. Note that in $\dfrac{1}{V}\dfrac{Y_t - Y_{t-1}}{Y_t - \alpha Y_{t-1}}$ a rise in velocity decreases the ratio of bank financing to the total change in assets and that a rise in the propensity to consume increases the dependence upon bank financing of investment. Therefore, autonomous or cyclically induced changes in these parameters can change the ratio of debt to equity financing, which can change the accelerator coefficient. In particular a rise in velocity tends to counteract the deterioration of firms' balance sheets in a business-cycle expansion financed by bank creation of money.

3. Autonomous or cyclically induced changes in the liquidity preference relation can change the dependence of an expansion upon changes in the money supply and therefore affect the ratio of bank debt to total assets of firms. If liquidity preference decreases, the excess of investment over *ex ante* saving can be financed by withdrawals from cash balances at lower interest rates than were previously ruling. Such an "autonomous" decrease in liquidity preference can, both by improving financing terms and by decreasing the dependence of business firms upon bank financing, raise the accelerator coefficient. A great stock-market boom, such as in the late 1920's, may be interpreted as reflecting a lowering of liquidity preferences; as a result business expansion could be financed with less reliance upon the banking system than otherwise.

Alternatively, an autonomous rise in liquidity preference may lead to the result that business borrowing from banks will increase the liquidity of households rather than finance investment. That is, a portion of business borrowing from banks ends up as "liquid hoards" of households. Such borrowing by business firms in excess of the difference between *ex ante* saving and realized investment will increase the rapidity with which firms' balance sheets deteriorate. An explosive accelerator process may be broken by such changes in liquidity preference.

Such changes in liquidity preference have been labeled autonomous. There exist plausible mechanisms by which the upward movement of an explosive accelerator process would lead to a fall in liquidity preference. However, there do not exist equally plausible mechanisms by which a rise in liquidity preference can be considered as endogenous during an expansion. During a downswing there exists a plausible mechanism which can raise the liquidity preference of households. This can force a deterioration of firms' balance sheets, and thereby, through its effect upon the accelerator coefficient, a further fall in investment. There does not seem to be any endogenous factor which would lead to a fall in liquidity preference on a downswing. Changes in liquidity preference seem to be destabilizing.

IV. *Policy Implications*

Let us asume that the policy goal is steady growth at a stable price level. The policy measures to be used are monetary policy, which in the language of this paper means to choose a monetary system, and fiscal policy. It has been shown that steady growth requires a money supply that increases at a geometric rate: but that a too rapidly growing money supply results in rapid price inflation and that a too slowly growing money supply results in a downturn of income.

The smallest self-sustaining rate of growth of income is equal to the smaller root of the accelerator process, μ_2. If productive capacity can also grow at this rate, then the policy goal of growth without inflation is attainable. If the rate of growth of income is greater than the maximum possible rate of growth of productive capacity, the policy goal is not attainable. In the latter case, we assume that steady growth accompanied by secular inflation will be chosen in preference to a constant price level and intermittent growth. The policy goal therefore becomes steady growth with a minimum rate of secular inflation.

If the policy-makers prize steady growth and abhor falling income, and if secular inflation is accepted as the price that has to be paid for growth, then the policy-makers would be able to "play it safe" by allowing the actual rate of growth of the money supply to be greater than the minimum self-sustainable rate of growth of income. That is, the policy-makers would accept some unnecessary inflation in order to be on the safe side in maintaining full employment.

For a given consumption coefficient, the greater the rate of growth of the money supply, the greater the ratio of bank debt to debt and equities to households in the balance sheets of firms. Therefore the greater the rate of increase in the money supply, the greater the chance that induced investment will decrease because of the unsatisfactory nature of firms' balance sheets. Two policy measures which can counteract this effect are: (1) an interest rate policy designed to keep velocity greater than one; (2) a fiscal policy designed to increase the money supply without increasing business debt to banks.

It was shown that if income velocity is greater than one and if the money supply is being increased by business borrowing from banks, the net increase in business borrowing from banks will be smaller than the difference between realized investment and *ex ante* saving. In order to achieve this result bank financing of business must be at a high enough interest rate to keep income velocity greater than one. But the accelerator coefficient also depends upon the interest rate. Thus if the monetary policy designed to keep income velocity greater than one is carried too far the accelerator coefficient will fall and the self-sustained growth will be interrupted.

To keep interest rates at a given level, the central bank must be willing to supply reserves to commercial banks, in response to commercial banks' demands, without limit at a fixed rediscount rate. Therefore the rediscount rate seems the appropriate tool of central bank policy.

Nevertheless if the money supply can increase only by business borrowing from banks, a ratio of debt to equities in business balance sheets can result which will lead to a decline in induced investment. Government deficits financed by borrowing from banks result in an increase in the money supply without any corresponding increase in business debt. If interest rates are such that velocity is greater than one, debts and equities to households will be substituted for debts to banks in the business firms' balance sheets. This is more conducive to steady growth than the situation in which all of the increase in the money supply required for steady growth is created in exchange for business debt. Therefore government deficit financing, even during a period of sustained growth and secularly rising prices, may be desirable in order to maintain the conditions for further growth.